APPENDIX B

SUMMARY OF ORBITAL MECHANICS RELEVANT TO REMOTE SENSING

Orbit selection and sensor characteristics are closely related to the strategy required to achieve the desired results. Different types of orbits are required to achieve continuous monitoring, repetitive coverage of different periodicities, global mapping, or selective imaging.

The vast majority of earth-orbiting, remote sensing satellites use circular orbits. Planetary orbiters usually have elliptical orbits, which are less taxing on the spacecraft orbital propulsion system and combine some of the benefits of high and low orbits, thus allowing a broader flexibility to achieve multiple scientific objectives.

B-1 CIRCULAR ORBITS

B-1-1 General Characteristics

A spacecraft will remain in a circular orbit around a spherically homogeneous planet if the inward gravitational force F_g is exactly cancelled by the outward centrifugal force F_c. These two forces are given by

$$F_g = mg_s\left(\frac{R}{r}\right)^2 \tag{B-1}$$

and

$$F_c = \frac{mv^2}{r} = m\omega^2 r \tag{B-2}$$

where g_s is the gravitational acceleration at the planet's surface, R is the planet radius of the planet, r is the orbit radius ($r = R + h$), h is the orbit altitude, v is the spacecraft linear velocity, and ω is the spacecraft angular velocity. For these two forces to be equal, the spacecraft linear velocity has to be

$$v = \sqrt{\frac{g_s R^2}{r}} \tag{B-3}$$

The orbital period T of the circular orbit is then

$$T = \frac{2\pi r}{v} = 2\pi r \sqrt{\frac{r}{g_s R^2}} \tag{B-4}$$

In the case of the Earth, $g_s = 9.81$ m/sec^2 and $R \approx 6380$ km. The orbital period T and linear velocity v are shown in Figure B-1 as a function of altitude h. For instance, at $h = 570$ km, $v \approx 7.6$ km/sec and $T = 1$ hr 36 min. In such an orbit, the spacecraft will orbit the Earth exactly 15 times per day. Table B-1 gives an illustrative example for the case of some planetary orbits.

In some cases, one might be interested in calculating the orbit altitude for a given orbital period. The orbit altitude can be found by inverting Equation (B-4):

$$h = \left[\frac{g_s R^2 T^2}{4\pi^2}\right]^{1/3} - R \tag{B-5}$$

The orientation of the orbit in space is specified in relation to the Earth's equatorial plane and the vernal equinox. The angle between the orbital plane and the equatorial plane is the orbital inclination I (see Figure B-2a). The angle between the vernal equinox (direction in space defined by the Earth–Sun line at the time of day–night equality) and the node line (intersection of the orbit and equatorial plane) is the orbital node longitude Ω.

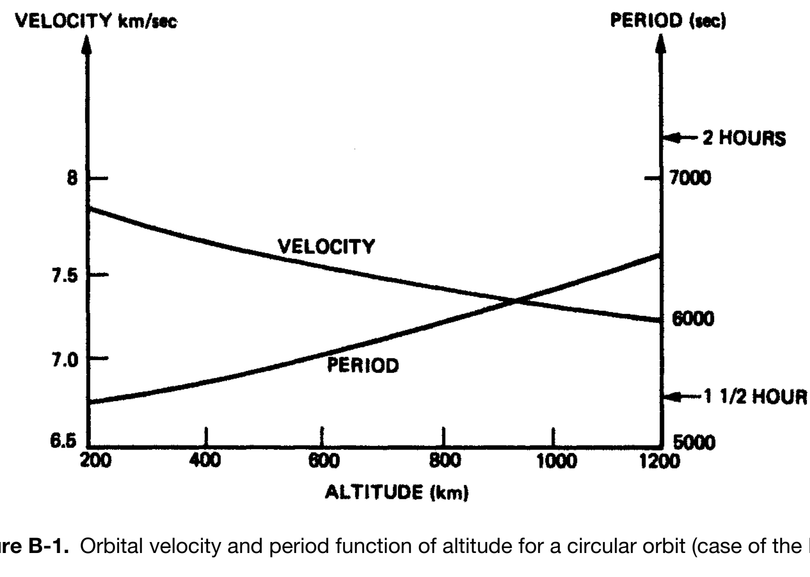


Figure B-1. Orbital velocity and period function of altitude for a circular orbit (case of the Earth).

TABLE B-1. Orbital Velocity and Period for Circular Orbits around Some of the Planets

Planet	Radius R (km)	Surface gravity g_s (m/sec^2)	Orbit altitude $h = 0.2R$		Orbit altitude $h = 0.5R$	
			v (km/sec)	T (min)	v (km/sec)	T (min)
Mercury	2440	3.63	2.7	113	2.4	158
Venus	6050	8.83	6.7	114	5.9	159
Earth	6380	9.81	7.2	111	6.4	155
Moon	1710	1.68	1.55	139	1.38	194
Mars	3395	3.92	3.3	128	3.0	179
Jupiter	71,500	25.9	39.3	229	35.0	320
Saturn	60,000	11.38	23.8	316	21.2	442

The largest variation in the orbit orientation is usually due to precession, which is the rotation of the orbit plane around the polar axis and is primarily due to the Earth's oblateness. The resulting rate of change of the nodal longitude Ω is approximated by:

$$\frac{d\Omega}{dt} = -\frac{3}{2} J_2 R^3 \sqrt{g_s} \frac{\cos I}{r^{7/2}} \tag{B-6}$$

where $J_2 = 0.00108$ is the coefficient of the second zonal harmonic of the Earth's geopotential field. Figure B-3 shows the relationship between $d\Omega/dt$, I, and h for the Earth. In the case of a polar orbit (i.e., $I = 90°$) the precession is zero because the orbit plane is coincident with the axis of symmetry of the Earth.

B-1-2 Geosynchronous Orbits

An important special case of the circular orbit is the geosynchronous orbit. This corresponds to the case in which the orbit period T is equal to the sidereal day, the sidereal day being the planet rotation period relative to the vernal equinox. Replacing T in Equation

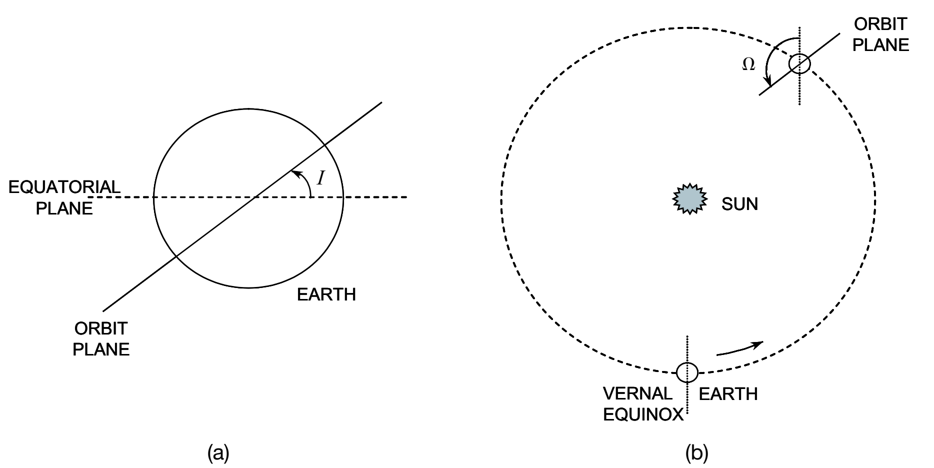


Figure B-2. Orientation of the orbit in space.

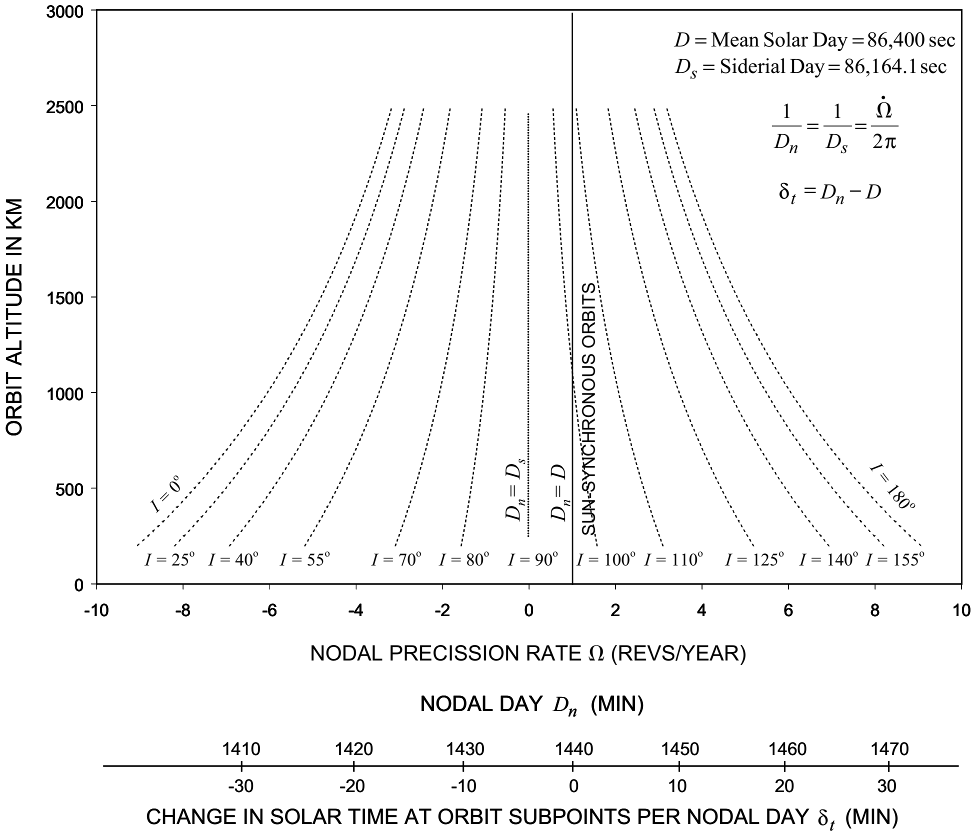


Figure B-3. Satellite orbit precession as a function of orbital altitude *h* and inclination *I*.

(B-5) by the length of the Earth's sidereal day 364/365 × 24 hr = 86,164 sec, we find that the radius of an Earth-geosynchronous circular orbit is $r \approx 42{,}180$ km, which corresponds to an altitude of 35,800 km. For a ground observer, a satellite at the geosynchronous altitude will appear to trace a figure-eight shape of latitude range $\pm I$ (see Fig. B-4). If the orbit is equatorial ($I = 0$), the satellite appears to be fixed in the sky. The orbit is then called geostationary. Geosynchronous orbits can also be elliptical. In this case, a distorted figure eight is traced relative to an Earth fixed frame (Figure B-5).

B-1-3 Sun-Synchronous Orbit

If the orbit precession exactly compensates for the Earth's rotation around the sun, the orbit is sun-synchronous (see Fig. B-3). Such an orbit provides a constant node-to-sun angle, and the satellite passage over a certain area occurs at the same time of the day.

All Landsat satellites are in a near-polar sun-synchronous orbit. Referring to Figure B-3, the Landsat orbit altitude of approximately 700 km corresponds to a sun-synchronous orbit inclination of approximately 98°. [Note from Equation (B-6) that sun-synchronous orbits require inclination angles larger than 90 degrees.]

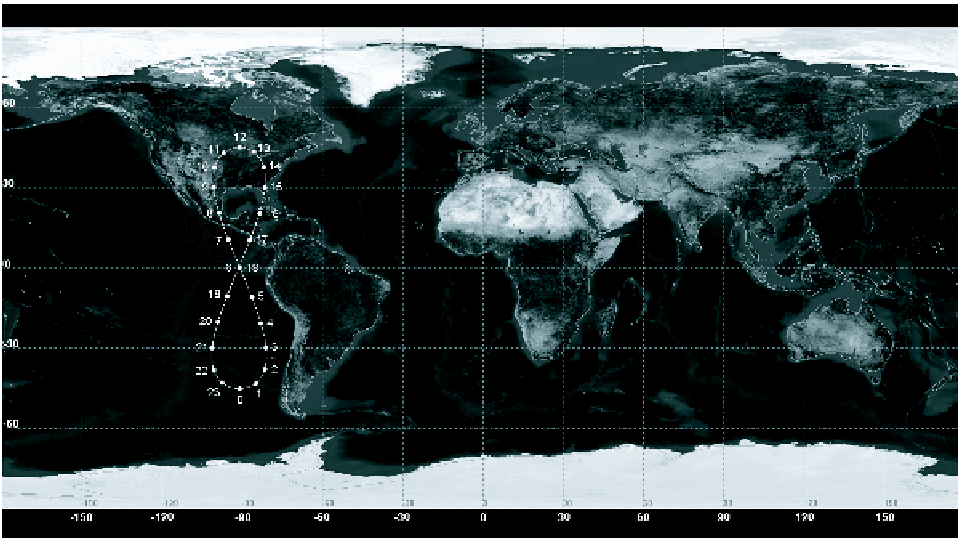


Figure B-4. Surface trace of a circular geosynchronous orbit with 45° inclination.

B-1-4 Coverage

The orbit nadir trace is governed by the combined motion of the planet and the satellite. Let us first assume that the planet is nonrotating. The nadir will resemble a sine wave on the surface map (Figure B-6*a*), with its great circle path developing a full 360° period (longitude) and covering latitudes between $\pm I$. Adding the planet's rotation causes a steady creep of the trace at a rate proportional to the ratio of the orbital motion angular rate to the planet's rotational rate (Fig. B-6*b*).

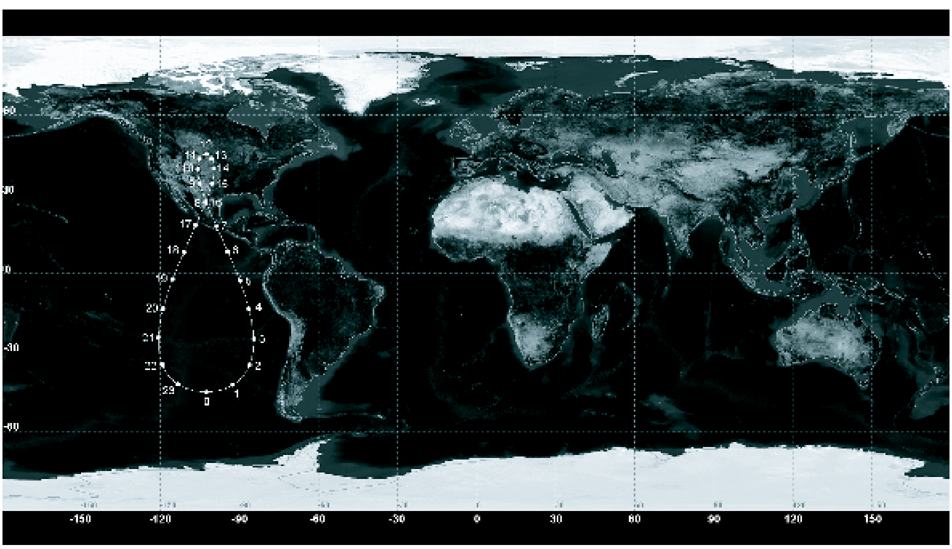


Figure B-5. Surface trace of an elliptical geosynchronous orbit with 45° inclination and 0.1 ellipticity.

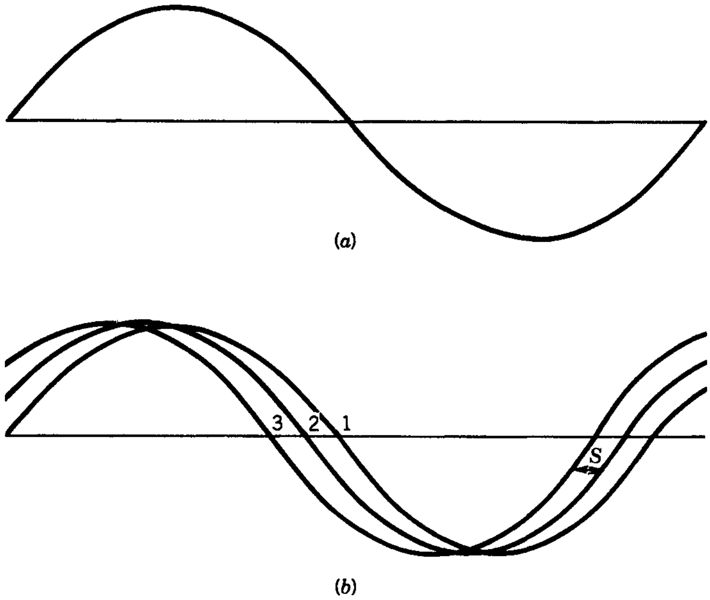


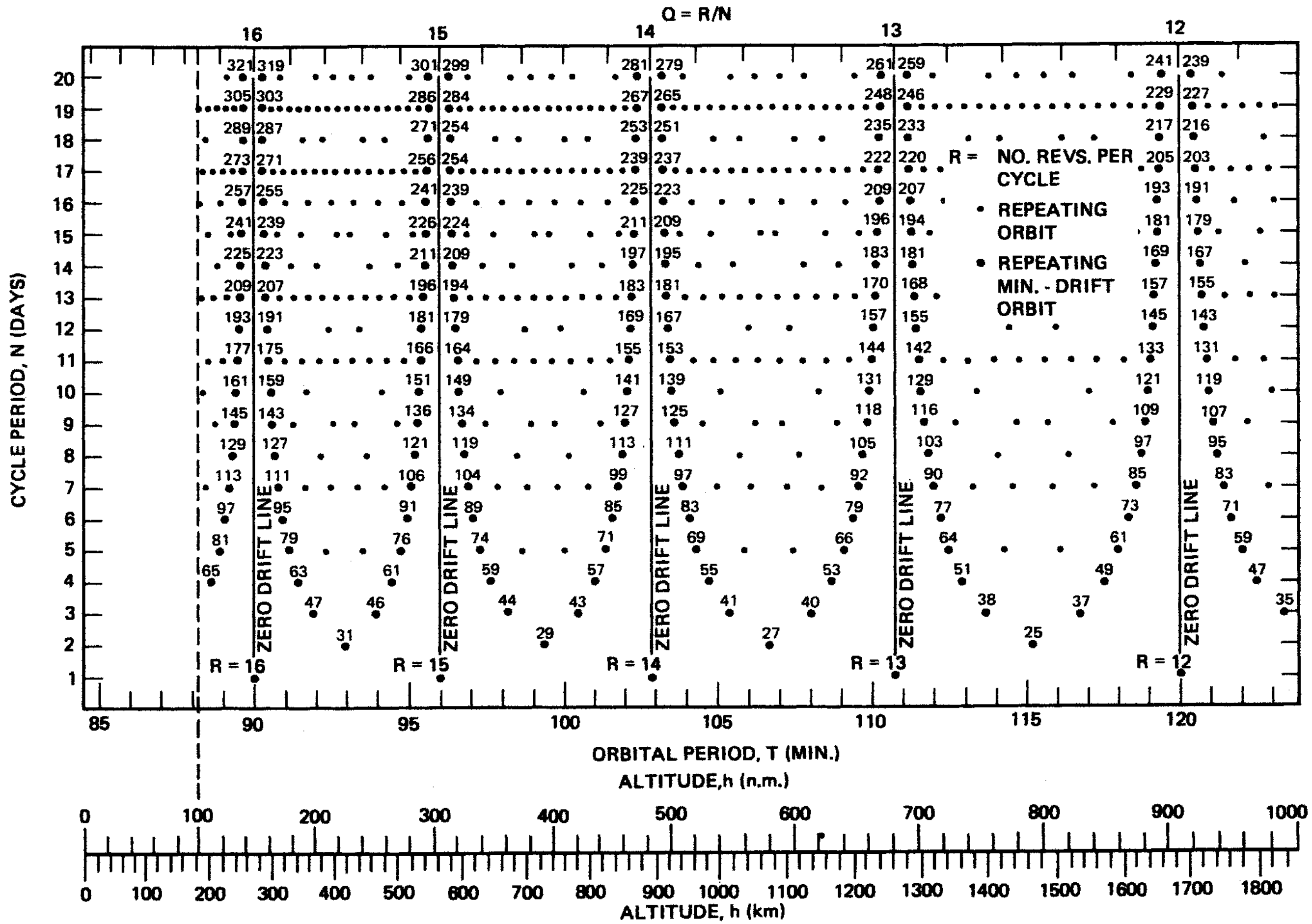
Figure B-6. (*a*) Orbit nadir trace on a nonrotating planet. (*b*) Orbit trace on a rotating planet. *S* is the orbit step that corresponds to the displacement of the equatorial crossing on successive orbits.

The orbit step S is the longitudinal difference between two consecutive equatorial crossings. If S is such that

$$S = 360\frac{N}{L}$$

where N and L are integer numbers, then the orbit coverage is repetitive. In this case, the satellite makes L revolutions as the planet makes N revolutions. N is thus the cycle period (sometimes called the orbit repeat period) and L is the number of revolutions per cycle. If $N = 1$, then there is an exact daily repeat of the coverage pattern. If $N = 2$, the repeat is every other day, and so on. Figure B-7 shows the relationship between N, L, the orbital period, and the orbital altitude for sun-synchronous orbits. It clearly shows that there is a wide array of altitude choices to meet any repetitive coverage requirement.

A wide range of coverage scenarios can be achieved by selecting the appropriate orbit. Figure B-8 gives examples of four different orbital altitudes, which would allow a 16-day repeat orbit, multiple viewing of a certain area on the surface, and complete coverage with a sensor having a field of view of about 178 km. The main difference is the coverage strategy. The 542 km (241 orbits in 16 days) and 867 km (225 orbits in 16 days) orbits have a drifting coverage, that is, the orbits on successive days allow mapping of contiguous strips. The 700 km orbit (233 orbits in 16 days) has a more dispersed coverage strategy, in which the second day strip is almost halfway in the orbital step. The 824 km orbit (227 orbits in 16 days) has a semidrifting coverage strategy, in which every five days the orbit drifts through the whole orbital strip, leaving gaps that are filled up on later orbits. The Landsat satellites complete 233 orbits in a 16 day cycle, corresponding to the coverage shown for 700 km altitude orbit.



NOTE: ABSCISSA SCALES (T AND h) ARE CORRECT FOR SUN-SYNCHRONOUS ORBITS AND APPROXIMATE FOR OTHERS.

Figure B-7. Periodic coverage patterns as a function of altitude for sun-synchronous orbits (case of Earth).

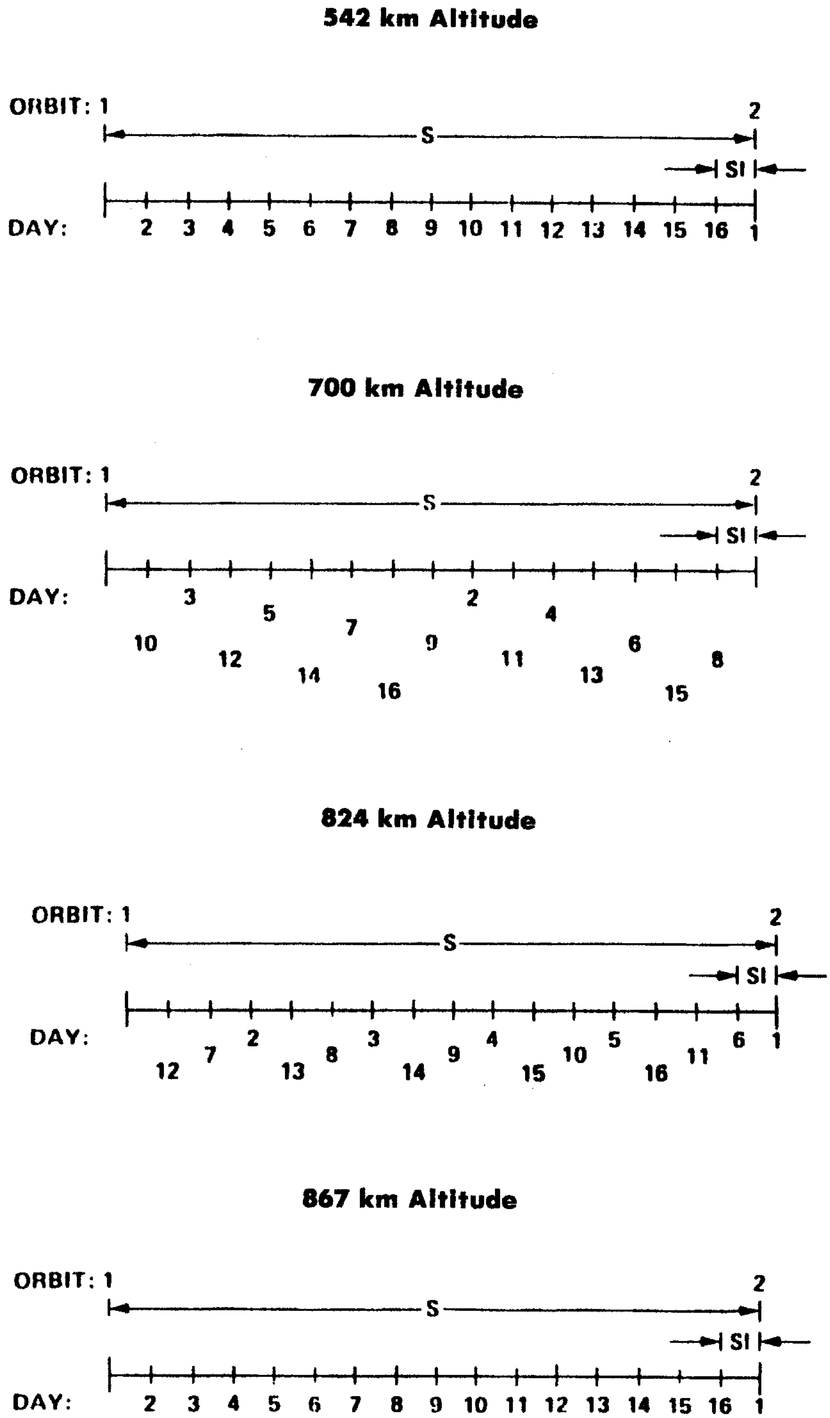


Figure B-8. Coverage scenario of four different orbital altitudes that provide an almost 16 day repeat orbit.

B-2 ELLIPTICAL ORBITS

Elliptical orbits are most commonly used for planetary orbiters. Such an orbit is desired to accommodate a wide variety of scientific objectives. For instance, high-resolution imaging, which requires low altitude, can be conducted near periapsis, whereas global meteorological monitoring can be done from near apoapsis, where the spacecraft stays for a longer time. The change in the satellite altitude will also allow the in situ measurement of the different regions of the planet's surroundings. In addition, the energy required to slow down a spacecraft to capture it in a circular orbit of altitude h_c is higher than the energy required for an elliptical orbit with a periapsis altitude h_c.

The characteristics of an elliptical orbit are derived from the solution of the two-body problem. The two most known characteristics are:

1. The radius vector of the satellite sweeps over an equal area in an equal interval of time.
2. The square of the orbital period is proportional to the cube of the mean satellite-to-planet-center distance.

The orbit is defined by

$$r = \frac{a(1 - e^2)}{1 + e\cos\theta} \tag{B-7a}$$

and

$$T = 2\pi\sqrt{\frac{a^3}{g_s R^2}} \tag{B-7b}$$

where r, θ are the satellite polar coordinates, a is the semimajor axis, and e is the orbit ellipticity (see Fig. B-9). By taking different combinations of values for a, e, and inclination, a wide variety of coverage strategies can be achieved. For example, Figure B-10 shows the ground trace for an elliptical orbit with $e = 0.75$, a 12 hour orbit ($a \approx 26{,}600$ km), and an

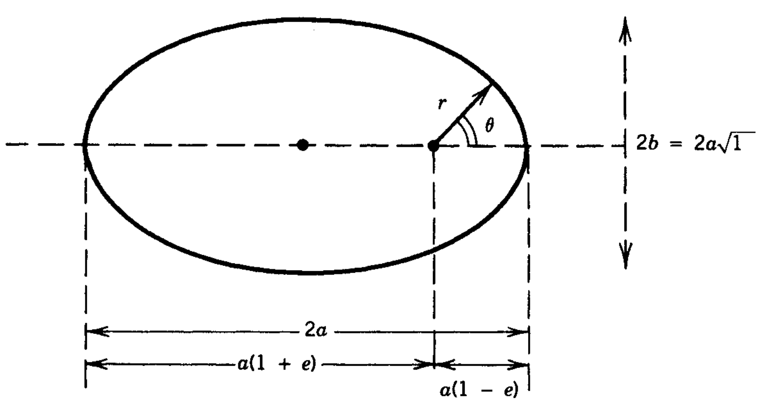


Figure B-9. Elliptical orbit.

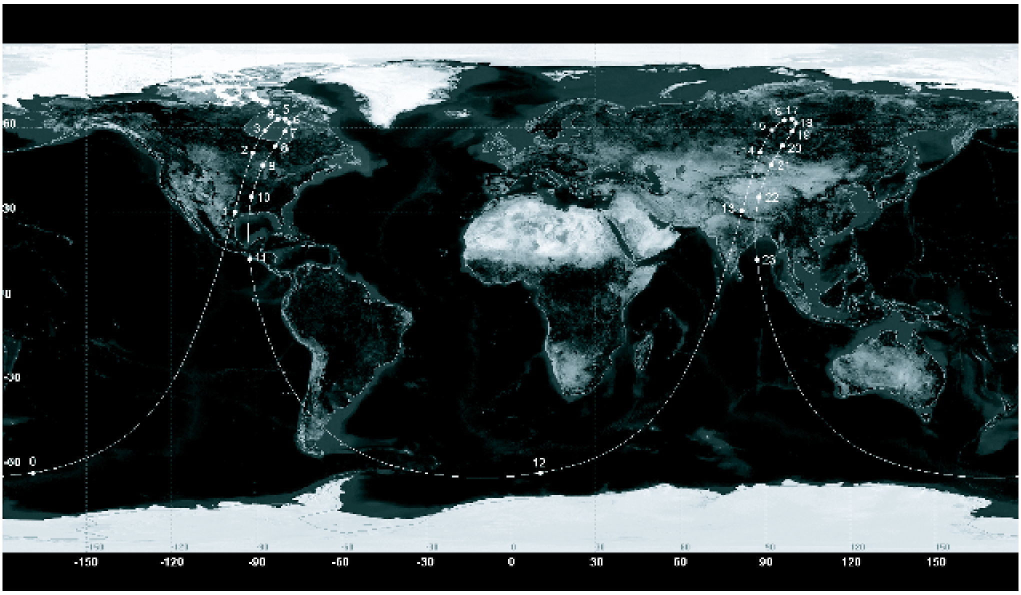


Figure B-10. Ground trace for a 12 hour elliptical orbit with an ellipticity of 0.7 and inclination of 63.4°.

inclination of 63.4°. It shows that the satellite spends most of the time over Europe and the north-central Pacific and only a small amount of time over the rest of the world. This is an example of a Molniya orbit, which has the property that rate of change of the orbit perigee is zero. For the Earth, this happens for inclinations of 63.4° and 116.6°.

B-3 ORBIT SELECTION

There are a number of factors that influence the selection of an orbit most appropriate for a specific remote sensing application. Some of these factors for an Earth orbiter are as follows:

To minimize Earth atmospheric drag—$h > 200$ km
Global coverage for Earth observation—polar or near-polar orbit
Astronomical/planetary observations from Earth orbit—equatorial orbit
Constant illumination geometry (optical sensors)—sun-synchronous orbits
Thermal inertia observations—one-day pass and one-night pass over same area
To minimize radar sensors' power—prefers low altitude
To minimize gravity anomalies perturbation—high altitude
To measure gravity anomalies—low altitude
Continuous monitoring—geostationary or geosynchronous orbit

EXERCISES

B-1. Plot the orbital velocity, velocity of projection on the surface, and orbital period for circular orbits around the Earth at altitudes ranging from 200 km to 10,000 km. Repeat the same for the case of Mercury, Venus, and Mars.

B-2. Plot the orbital velocity and orbital period for circular orbits around Jupiter and Saturn at altitudes ranging from 60,000 to 200,000 km.

B-3. Plot the nodal precession rate of circular orbits around the Earth as a function of altitude ranging from 200 km to 2500 km for orbit inclinations of 0°, 40°, 60°, 80°, 90°, 100°, 120°, 140°, and 180°. Express the precession rate in radians/year or revs/year. Indicate on the plot the sun-synchronous orbits.

B-4. Calculate the altitude of geostationary orbits for Mercury, Venus, Mars, Jupiter, and Saturn. The rotation periods of these planets are 58.7 Earth-days, 243 Earth-days, 24 hr 37 min, 9 hr 51 min, and 10 hr 14 min, respective1y.

B-5. Let us assume that an Earth-orbiting sensor in polar circular orbit requires daily repeat (i.e. $N = 1$). Calculate the lowest three orbits which allow such a repeat coverage. Calculate the lowest three orbits for repeats every two days ($N = 2$) and every three days ($N = 3$).

B-6. A radar mission is designed to fly on the space shuttle with the aim of mapping as much of the Earth as possible in 10 days. Given the size and mass of the radar payload, the maximum inclination of the orbit is 57°, and the altitude range is 200 km to 250 km. Calculate the altitude of an orbit in this range that would repeat in exactly 10 days. Calculate the separation between orbit tracks along the equator.